Applications of $b^*g_*$-Closed Sets and $b^*g_*$-Functions in Topological Spaces

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Abstract: The aim of this paper is to introduce a new class of sets called $b^*g_*$-closed sets in topological spaces. We also introduce $b^*g_*$-continuous function, $b^*g_*$-irresolute function, $b^*g_*$-open map, $b^*g_*$-closed map and $b^*g_*$-homeomorphisms. Moreover we define some spaces based on $b^*g_*$-closed sets in topological spaces.

Keywords: $b^*g_*$-closed sets; $b^*g_*$-continuous; $b^*g_*$-irresolute; $\mathcal{J}_{b^*g_*}$-space; $*_nT_{1/2}$ ***-space; $*_nT_{***}$-space; $b^*g_*$-homeomorphisms

I. INTRODUCTION

N. Levine\([8]\) introduced generalized-closed sets in topological spaces and a class of topological space called $T_{1/2}$-spaces. D. Andrijevic\([1]\) introduced and investigated $b$-open sets. M. Vigneshwaran and R. Devi\([18]\) introduced $^*g_*$-closed sets in topological spaces. K. Ayswarya\([3]\) introduced and derived the properties of $s^*g_*$-closed sets. M. Vigneshwaran and K. Baby\([17]\) introduced $\beta^*g_*$-closed sets in topological spaces. K. Geetha and M. Vigneshwaran\([6]\) introduced and studied the properties of $p^*g_*$-closed sets in topological spaces.

In this paper, we introduce a new class of sets called $b^*g_*$-closed sets in topological spaces. By using this set, we have introduced $\mathcal{J}_{b^*g_*}$, $*_nT_{1/2}$ ***, $*_{b^*g_*}$, $*_nT_{***}$ and $*_nT_{***}$-spaces. We also introduce the notion of $b^*g_*$-continuous, $b^*g_*$-irresoluteness, $b^*g_*$-open map, $b^*g_*$-closed map and $b^*g_*$-homeomorphisms.

II. PRILIMINARIES

Throughout this paper $(X, \tau)$, $(Y, \sigma)$ and $(Z, \eta)$ represent topological space. For a subset $A$ of a space $(X, \tau)$, $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of $A$ and the interior of $A$ respectively. Let us recall the following definitions.

Definition: 2.1

A subset $A$ of a topological space $(X, \tau)$ is called,

- a semi-open set if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$,
- a pre-open set if $A \subseteq \text{int}(\text{cl}(A))$ and a pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$,
- an $\alpha$-open set if $A \subseteq \text{int}(\text{cl}(A))$ and an $\alpha$-closed set if $\text{cl}(\text{int}(A)) \subseteq A$,
- a semi-preopen set($\beta$-open) if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-preclosed set($\beta$-closed) if $\text{int}(\text{cl}(A)) \subseteq A$,
- a $b$-open set if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$ and $b$-closed set if $\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \subseteq A$.
Definition: 2.2
A subset $A$ of a topological space $(X, \tau)$ is called

- a $g$-closed set[8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$,
- a $\psi$-closed set[14] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $sg$-open in $(X, \tau)$,
- a $g^*$-closed set[13] if $d(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$,
- a $g^\#$-closed set[15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $ag$-open in $(X, \tau)$,
- a $g^\#s$-closed set[16] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $ag$-open in $(X, \tau)$,
- a $g^\#\alpha$-closed set[12] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g\alpha$-open in $(X, \tau)$,
- a $g^\#\psi$-closed set[14] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$,
- a $^*g\alpha$-closed set[18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g\alpha$-open in $(X, \tau)$,
- an $^*ag\alpha$-closed set[7] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^*g\alpha$-open in $(X, \tau)$,
- an $^*g\alpha$-closed[3] set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^*g\alpha$-open in $(X, \tau)$,
- a gsp-closed set[5] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X, \tau)$.

Definition: 2.3
A space $(X, \tau)$ is called

- a $T_b^\#$-space [16], if every $g^\#s$-closed set is closed,
- an $T_{1/2}^*\#$-space [18], if every $^*g\alpha$-closed set is closed,
- an $T_{1/2}^\#$-space [12], if every $g^\#\alpha$-closed set is $g^\*$-closed,
- a $T_{1/2}^*\$-space [13], if every $g^\*$-closed set is closed,
- an $T_{1/2}^*\$-space [7], if every $^*g\alpha$-closed set is closed,
- an $T_{1/2}^*\$-space [3], if every $s^*g\alpha$-closed set is closed,
- an $T_{1/2}^*\$-space [7], if every $^*g\alpha$-closed set is $^*g\alpha$-closed,
- a $^*T_{1/2}^*\$-space [3], if every $s^*g\alpha$-closed set is $^*g\alpha$-closed.

Definition: 2.4
A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- $g$-continuous[8] if $f^{-1}(V)$ is $g$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $^*g\alpha$-continuous[13] if $f^{-1}(V)$ is $^*g\alpha$-closed set of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $^*g\alpha$-continuous[18] if $f^{-1}(V)$ is $g\alpha$ -closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $^*g\alpha$-continuous[12] if $f^{-1}(V)$ is $^*g\alpha$-closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $g^\#s$-continuous[16] if $f^{-1}(V)$ is $g^\#s$-closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $^*g\psi$-continuous[14] if $f^{-1}(V)$ is $^*g\psi$-closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $^*ag\alpha$-continuous[7] if $f^{-1}(V)$ is $^*ag\alpha$ -closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
- $s^*g\alpha$-continuous[3] if $f^{-1}(V)$ is $s^*g\alpha$ -closed set in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

Notation 2.5:
For a space $(X, \tau)$, $C(X, \tau)$ denote the class of all closed subsets of $(X, \tau)$.

**III. BASIC PROPERTIES OF $b^*g\alpha$-CLOSED SETS**

We introduce the following definition.

Definition: 3.1
A subset $A$ of $(X, \tau)$ is called a $b^*g\alpha$-closed set if $bc(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^*g\alpha$-open in $(X, \tau)$.
Theorem: 3.2
Every closed set is b*ga-closed set.

Proof:
Let A ⊆ U, where U is *ga-open set in (X, τ). Since A is closed, cl(A) = A. But bcl(A) ⊆ cl(A) = A, which implies bcl(A) ⊆ U. Hence A is b*ga-closed.

The converse of the above theorem need not be true by the following example.

Example: 3.3
Let X = {a, b, c} with τ = {X, φ, {a}, {c}, {a, c}}. C(X, τ) = {X, φ, {b}, {a, b}, {b, c}} and b*ga C(X, τ) = {X, φ, {b}, {c}, {a, b}, {b, c}}. Here {a} and {c} are b*ga-closed but not closed.

Theorem: 3.4
Every b-closed set is b*ga-closed set.

Proof:
Let A ⊆ U, where U is *ga-open set in (X, τ). Since A is b-closed, bcl(A) = A. Therefore bcl(A) ⊆ U. Hence A is b*ga-closed.

The converse of the above theorem need not be true by the following example.

Example: 3.5
Let X = {a, b, c} with τ = {X, φ, {a}, {a, b}}. b C(X, τ) = {X, φ, {b}, {c}, {b, c}} and b*ga C(X, τ) = {X, φ, {b}, {c}, {a, c}}. Here {a, c} is b*ga-closed but not b-closed.

Theorem: 3.6
Every α-closed, semi-closed, pre-closed set is b*ga-closed set.

Proof:
Let A ⊆ U, where U is *ga-open set in (X, τ). Since A is an α-closed, semi-closed and a pre-closed set, then bcl(A) ⊆ αcl(A) = A, also bcl(A) ⊆ scl(A) = A, and also bcl(A) ⊆ pcl(A) = A, which implies bcl(A) ⊆ U. Therefore A is b*ga-closed.

The converse of the above theorem need not be true by the following example.

Example: 3.7
Let X = {a, b, c} with τ = {X, φ, {a}, {a, b}}. α C(X, τ) = {X, φ, {b}, {c}, {b, c}} = SC(X, τ) = PC(X, τ) and b*ga C(X, τ) = {X, φ, {b}, {c}, {b, c}, {a, c}}. Here {a, c} is b*ga-closed but not α-closed, semi-closed and pre-closed.

Theorem: 3.8
Every *ga-closed set is b*ga-closed set.

Proof:
Let A ⊆ U, where U is *ga-open set in (X, τ). Since every *ga-open set is ga-open, U is ga-open. Since A is *ga-closed in (X, τ), cl(A) ⊆ U. But bcl(A) ⊆ cl(A) ⊆ U, which implies bcl(A) ⊆ U. Therefore A is b*ga-closed.

The converse of the above theorem need not be true by the following example.

Example: 3.9
Let X = {a, b, c} with τ = {X, φ, {a}, {c}, {a, c}}. *ga C(X, τ) = {X, φ, {b}, {a, b}, {b, c}} and b*ga C(X, τ) = {X, φ, {a}, {b}, {c}, {a, b}, {b, c}}. Here {a} and {c} are b*ga-closed but not *ga-closed.

Theorem: 3.10
Every g#α-closed, g*ψ-closed and g#s-closed set is b*ga-closed set.

Proof:
Let A ⊆ U, where U is *ga-open set in (X, τ). Since every *ga-open set is g-open, U is g-open. Since A is g#α-closed in (X, τ), then acl(A) ⊆ U, but bcl(A) ⊆ acl(A) ⊆ U also A is g*ψ-closed in (X, τ) then ψcl(A) ⊆ U, but bcl(A) ⊆ ψcl(A) ⊆ U and also every *ga-open set is αg-open, U is αg-open. Since A is g#s-closed in (X, τ), scl(A) ⊆ U. But bcl(A) ⊆ scl(A) ⊆ U, which implies bcl(A) ⊆ U. Therefore A is b*ga-closed.

The converse of the above theorem need not be true by the following example.
We introduce the following definition.

Definition 3.11

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. $g^*\alpha C(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ and $b^*g^*\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. Here $\{a\}$ and $\{c\}$ are $b^*g^*\alpha$-closed but not $g^*\alpha$-closed, $g^*\psi$-closed and $g^*s$-closed.

Theorem 3.12

Every $g^*$-closed set is $b^*g^*\alpha$-closed.

Proof:

Let $A \subseteq U$, where $U$ is $*g^*\alpha$-open set in $(X, \tau)$. Since every $*g^*\alpha$-open set is $g$-open, $U$ is $g$-open. Since $A$ is $g^*$-closed in $(X, \tau)$, $cl(A) \subseteq U$. But $bcl(A) \subseteq cl(A) \subseteq U$, which implies $bcl(A) \subseteq U$. Therefore $A$ is $b^*g^*\alpha$-closed.

The converse of the above theorem need not be true by the following example.

Example 3.13

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. $g^* C(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ and $b^*g^* C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. Here $\{a\}$ and $\{c\}$ are $b^*g^*\alpha$-closed but not $g^*\alpha$-closed.

Theorem 3.14

Every $*g\alpha\alpha$-closed set is $b^*g^*\alpha$-closed.

Proof:

Let $A \subseteq U$, where $U$ is $*g\alpha\alpha$-open set in $(X, \tau)$. Since $A$ is $*g\alpha\alpha$-closed set, $\alpha cl(A) \subseteq U$. But $bcl(A) \subseteq \alpha cl(A) \subseteq U$, which implies $bcl(A) \subseteq U$. Therefore $A$ is $b^*g^*\alpha$-closed.

The converse of the above theorem need not be true by the following example.

Example 3.15

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. $b^*g^*\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}$ and $*g\alpha\alpha C(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$. Here $\{a\}$ and $\{b\}$ are $b^*g^*\alpha$-closed but not $*g\alpha\alpha$-closed.

Theorem 3.16

Every $s^*g\alpha\alpha$-closed set is $b^*g^*\alpha$-closed.

Proof:

Let $A \subseteq U$, where $U$ is $*g\alpha\alpha$-open set in $(X, \tau)$. Since $A$ is $s^*g\alpha\alpha$-closed set, $\alpha cl(A) \subseteq U$. But $bcl(A) \subseteq \alpha cl(A) \subseteq U$, which implies $bcl(A) \subseteq U$. Therefore $A$ is $b^*g^*\alpha$-closed.

The converse of the above theorem need not be true by the following example.

Example 3.17

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. $s^*g\alpha\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}$ and $s^*g\alpha\alpha C(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$. Here $\{a\}$ and $\{b\}$ are $b^*g^*\alpha$-closed but not $s^*g\alpha\alpha$-closed.

Theorem 3.18

Every $b^*g^*\alpha$-closed set is gsp-closed set.

Proof:

Let $A \subseteq U$, where $U$ is open set in $(X, \tau)$. Since every open set is $*g\alpha\alpha$-open, $U$ is $*g\alpha\alpha$-open. Since $A$ is $b^*g^*\alpha$-closed set, $bcl(A) \subseteq U$. But $spcl(A) \subseteq cl(A) \subseteq U$, which implies $spcl(A) \subseteq U$. Therefore $A$ is $gsp$-closed.

The converse of the above theorem need not be true by the following example.

Example 3.19

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{c\}, \{a, c\}\}$. $b^*g^*\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}$ and $gsp C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Here $\{b\}$ and $\{a, b\}$ are gsp-closed but not $b^*g^*\alpha$-closed.

**IV. APPLICATIONS OF $b^*g^*\alpha$-CLOSED SETS**

We introduce the following definition.

Definition 4.1

A space $(X, \tau)$ is called a $T_{b^*g^*\alpha}$-space if every $b^*g^*\alpha$-closed set is closed.

The following theorem gives the characterization of $T_{b^*g^*\alpha}$-space.
Theorem 4.2

Every $T_{b^*ga}$-space is a $T_{1/2^{**}}$-space.

Proof:

Let $A$ be a $g^*gt$-closed set of $(X, \tau)$. Since every $g^*gt$-closed set is $b^*gt$-closed, $A$ is $b^*gt$-closed. Since $X$ is $T_{b^*ga}$-space, $A$ is closed. Therefore $(X, \tau)$ is $T_{1/2^{**}}$-space.

The converse of the above theorem need not be true by the following example.

Example 4.3

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, c\}\}$. $C(X, \tau) = \{X, \phi, \{b\}, \{a, c\}\}; g^*at C(X, \tau) = \{X, \phi, \{b\}, \{a, c\}\}$

and $b^*gt C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here $(X, \tau)$ is $T_{1/2^{**}}$-space but not $T_{b^*ga}$-space.

Since $\{a\}, \{c\}, \{a, b\}$ and $\{b, c\}$ are $b^*gt$-closed but not closed.

Theorem 4.4

Every $T_{b^*ga}$-space is a $T_b^*$-space.

Proof:

Let $A$ be a $gs^*$-closed set of $(X, \tau)$. Since every $gs^*$-closed set is $b^*gt$-closed, $A$ is $b^*gt$-closed. Since $X$ is $T_{b^*ga}$-space, $A$ is closed. Therefore $(X, \tau)$ is $T_b^*$-space.

The converse of the above theorem need not be true by the following example.

Example 4.5

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{c\}, \{a, b\}\}$. $C(X, \tau) = \{X, \phi, \{c\}, \{a, b\}\}; gs^* C(X, \tau) = \{X, \phi, \{c\}, \{a, b\}\}$

and $b^*gt C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here $(X, \tau)$ is $T_b^*$-space but not $T_{b^*ga}$-space. Since $\{a\}, \{b\}, \{a, c\}$ and $\{b, c\}$ are $b^*gt$-closed but not $gs^*$-closed.

Theorem 4.6

Every $T_{b^*ga}$-space is a $T_{1/2}^{**}$-space.

Proof:

Let $A$ be a $gs^*$-closed set of $(X, \tau)$. Since every $gs^*$-closed set is $b^*gt$-closed, $A$ is $b^*gt$-closed. Since $X$ is $T_{b^*ga}$-space, $A$ is closed. Since every closed set is $g$-closed, $A$ is $g$-closed. Therefore $(X, \tau)$ is $T_{1/2}^{**}$-space.

The converse of the above theorem need not be true by the following example.

Example 4.7

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. $C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}; g^*at C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$

and $b^*gt C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here $(X, \tau)$ is $T_{1/2}^{**}$-space but not $T_{b^*ga}$-space.

Since $\{b\}, \{c\}, \{a, b\}$ and $\{a, c\}$ are $b^*gt$-closed but not $gs^*$-closed.

Theorem 4.8

Every $T_{b^*ga}$-space is a $T_{1/2}^{*}$-space.

Proof:

Let $A$ be a $g^*$-closed set of $(X, \tau)$. Since every $g^*$-closed set is $b^*gt$-closed, $A$ is $b^*gt$-closed. Since $X$ is $T_{b^*ga}$-space, $A$ is closed. Therefore $(X, \tau)$ is $T_{1/2}^{*}$-space.

The converse of the above theorem need not be true by the following example.

Example 4.9

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. $C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}; g C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$

and $b^*gt C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here $(X, \tau)$ is $T_{1/2}^{*}$-space but not $T_{b^*ga}$-space. Since $\{b\}, \{c\}, \{a, b\}$ and $\{a, c\}$ are $b^*gt$-closed but not $g^*$-closed.
Theorem: 4.10

Every \( cT_{b*gz} \)-space is \( *T_{1/2}^{**} \)-space.

Proof:

Let \( A \) be \( *\text{gz} \)-closed set of \((X, \tau)\). Since every \( *\text{gz} \)-closed set is \( b*gz \)-closed, \( A \) is \( b*gz \)-closed. Since \( X \) is \( cT_{b*gz} \)-space, \( A \) is closed. Therefore \((X, \tau)\) is \( *T_{1/2}^{**} \)-space.

The converse of the above theorem need not be true by the following example.

Example: 4.11

Let \( X = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\} \). \( C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}, \{a, c\}\} \); \( b*gz \) \( C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\} \) and \( *\text{gz} \) \( C(X, \tau) = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\} \). Here \((X, \tau)\) is \( *T_{1/2}^{**} \)-space but not \( cT_{b*gz} \)-space. Since \( \{a\} \) and \( \{b\} \) are \( b*gz \)-closed but not \( *\text{gz} \)-closed.

We introduce the following definition.

Definition: 4.12

A space \((X, \tau)\) is called \( *_{b}T_{1/2}^{***} \)-space if every \( b*gz \)-closed set is \( *\text{gz} \)-closed.

The following theorems give the characterizations of \( *_{b}T_{1/2}^{***} \)-space.

Theorem: 4.13

Every \( *_{b}T_{1/2}^{***} \)-space is \( *_{s}T_{1/2}^{***} \)-space.

Proof:

Let \( A \) be \( *\text{gz} \)-closed set of \((X, \tau)\). Since every \( *\text{gz} \)-closed set is \( b*gz \)-closed, \( A \) is \( b*gz \)-closed. Since \( X \) is \( *_{b}T_{1/2}^{***} \)-space, \( A \) is \( *\text{gz} \)-closed. Therefore \((X, \tau)\) is \( *_{s}T_{1/2}^{***} \)-space.

The converse of the above theorem need not be true by the following example.

Example: 4.14

Let \( X = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\} \). \( C(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\} \); \( b*gz \) \( C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\} \) and \( *\text{gz} \) \( C(X, \tau) = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\} \). Here \((X, \tau)\) is \( *_{s}T_{1/2}^{***} \)-space but not \( *_{b}T_{1/2}^{***} \)-space. Since \( \{a\} \) and \( \{b\} \) are \( b*gz \)-closed but not \( *\text{gz} \)-closed.

Theorem: 4.15

Every \( *_{b}T_{1/2}^{***} \)-space is \( *_{s}T_{1/2}^{***} \)-space.

Proof:

Let \( A \) be \( s*\text{gz} \)-closed set of \((X, \tau)\). Since every \( s*\text{gz} \)-closed set is \( b*gz \)-closed, \( A \) is \( b*gz \)-closed. Since \( X \) is \( *_{b}T_{1/2}^{***} \)-space, \( A \) is \( *\text{gz} \)-closed. Therefore \((X, \tau)\) is \( *_{s}T_{1/2}^{***} \)-space.

The converse of the above theorem need not be true by the following example.
Example: 4.16

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. $C(X, \tau) = \{X, \phi, \{c\}\}$; $b^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}$. $s^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{c\}, \{a, c\}\}$ and $s^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{c\}, \{a, c\}\}$.

We introduce the following definition.

**Definition: 4.17**

A space $(X, \tau)$ is called $^{*}_{b}T_{1/2}$-space if every $b^{*}\text{g}_{\alpha}$-closed set is $g^{*}$-closed.

**Theorem: 4.18**

Every $^{*}_{b}T_{1/2}$-space is $^sT_{1/2}$-space.

**Proof:**

Let $A$ be a $g^{*}\alpha$-closed set of $(X, \tau)$. Since $X$ is $^{*}_{b}T_{1/2}$-space, $A$ is $b^{*}\text{g}_{\alpha}$-closed. Since every $b^{*}\text{g}_{\alpha}$-closed set is $*\text{g}_{\alpha}$-closed, $A$ is $*\alpha\text{g}_{\alpha}$-closed. Therefore $(X, \tau)$ is $^{*}_{b}T_{1/2}$-space.

The converse of the above theorem need not be true by the following example.

**Example: 4.19**

Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. $C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$; $b^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. $s^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$ and $s^{*}\text{g}_{\alpha}C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$.

We introduce the following definitions.

**Definition: 4.20**

(i) A space $(X, \tau)$ is called $^{*}_{b}T_{***}$-space if every $b^{*}\text{g}_{\alpha}$-closed set is $*\text{g}_{\alpha}$-closed.

(ii) A space $(X, \tau)$ is called $^{*}_{b}sT_{***}$-space if every $b^{*}\text{g}_{\alpha}$-closed set is $s^{*}\text{g}_{\alpha}$-closed.

**Theorem: 4.21**

(i) Every $^{*}_{b}T_{1/2}$-space is $^{*}_{b}T_{***}$-space.

(ii) Every $^{*}_{b}T_{1/2}$-space is $^{*}_{b}sT_{***}$-space.

**Proof:**

(i) Let $A$ be a $b^{*}\text{g}_{\alpha}$-closed set of $(X, \tau)$. Since $X$ is $^{*}_{b}T_{1/2}$-space, $A$ is $*\text{g}_{\alpha}$-closed. Since every $*\text{g}_{\alpha}$-closed set is $*\text{g}_{\alpha}$-closed, $A$ is $*\text{g}_{\alpha}$-closed. Therefore $(X, \tau)$ is $^{*}_{b}T_{***}$-space.

(ii) Let $A$ be a $b^{*}\text{g}_{\alpha}$-closed set of $(X, \tau)$. Since $X$ is $^{*}_{b}T_{1/2}$-space, $A$ is $*\text{g}_{\alpha}$-closed. Since every $*\text{g}_{\alpha}$-closed set is $s^{*}\text{g}_{\alpha}$-closed, $A$ is $s^{*}\text{g}_{\alpha}$-closed. Therefore $(X, \tau)$ is $^{*}_{b}sT_{***}$-space.
The converse of the above theorem need not be true by the following example.

Example: 4.22

Let \( X = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a\}, \{b, c\}\} \). \( C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}; b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}; \) 
\[
\begin{align*}
& \{a, b\} \{b, c\}, \{a, c\}; \quad \ast_{g_{\alpha}} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \\
& \{a, b\} \{b, c\}, \{a, c\}; \quad \ast_{2g_{\alpha}} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \\
& \{a, b\} \{b, c\}, \{a, c\}; \quad \ast_{3g_{\alpha}} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}.
\end{align*}
\]

V. \( b^*g_{\alpha} \)-CONTINUOUS and IRRESOLUTE FUNCTIONS

Definition: 5.1

A function \( f: (X, \tau) \to (Y, \sigma) \) is called \( b^*g_{\alpha} \)-continuous if \( f^{-1}(V) \) is a \( b^*g_{\alpha} \)-closed set of \( (X, \tau) \) for every closed set \( V \) of \( (Y, \sigma) \).

Theorem: 5.2

Every continuous map is \( b^*g_{\alpha} \)-continuous map.

Proof:

Let \( V \) be a closed set in \( (Y, \sigma) \). Since \( f \) is continuous, \( f^{-1}(V) \) is closed in \( (X, \tau) \). But every closed set is \( b^*g_{\alpha} \)-closed set. Hence \( f^{-1}(V) \) is \( b^*g_{\alpha} \)-closed set in \( (X, \tau) \). Hence every continuous map is \( b^*g_{\alpha} \)-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.3

\( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \). \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \). Let \( f: (X, \tau) \to (Y, \sigma) \) be defined as, \( f(a) = a, f(b) = c \) and \( f(c) = b \). Then \( f \) is \( b^*g_{\alpha} \)-continuous but not \( *g_{\alpha} \)-continuous. Since \( f^{-1}(a, c) = \{a, b\} \) is not closed in \( (X, \tau) \).

Theorem: 5.4

Every \( *g_{\alpha} \)-continuous map is \( b^*g_{\alpha} \)-continuous map.

Proof:

Let \( V \) be a closed set in \( (Y, \sigma) \). Since \( f \) is \( *g_{\alpha} \)-continuous, \( f^{-1}(V) \) is a \( *g_{\alpha} \)-closed in \( (X, \tau) \). But every \( *g_{\alpha} \)-closed set is \( b^*g_{\alpha} \)-closed set. Hence \( f^{-1}(V) \) is a \( b^*g_{\alpha} \)-closed set in \( (X, \tau) \). Hence \( f \) is \( b^*g_{\alpha} \)-continuous. Hence every \( *g_{\alpha} \)-continuous map is \( b^*g_{\alpha} \)-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.5

\( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \). Let \( f: (X, \tau) \to (Y, \sigma) \) be defined as, \( f(a) = b, f(b) = a \) and \( f(c) = c \). Then \( f \) is \( b^*g_{\alpha} \)-continuous but not \( *g_{\alpha} \)-continuous. Since \( f^{-1}(a, c) = \{b, c\} \) is not closed in \( (X, \tau) \).

Theorem: 5.6

Every \( g_{\alpha} \)-continuous map is \( b^*g_{\alpha} \)-continuous.

Proof:

Let \( V \) be a closed set in \( (Y, \sigma) \). Since \( f \) is \( g_{\alpha} \)-continuous map. Therefore \( f^{-1}(V) \) is a \( g_{\alpha} \)-closed set in \( (X, \tau) \). But every \( g_{\alpha} \)-closed set is \( b^*g_{\alpha} \)-closed set. Hence \( f^{-1}(V) \) is a \( b^*g_{\alpha} \)-closed set in \( (X, \tau) \). Hence \( f \) is \( b^*g_{\alpha} \)-continuous. Hence every \( g_{\alpha} \)-continuous map is \( b^*g_{\alpha} \)-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.7

\( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \) and \( b^*g_{\alpha} C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\} \). Let \( f: (X, \tau) \to (Y, \sigma) \) be defined as, \( f(a) = a, f(b) = b \) and \( f(c) = c \). Then \( f \) is \( b^*g_{\alpha} \)-continuous but not \( g_{\alpha} \)-continuous. Since \( f^{-1}(a, b) = \{a, b\} \) is not \( g_{\alpha} \)-closed in \( (X, \tau) \).
Theorem: 5.8
Every g#s-continuous map is b*gα-continuous map.

Proof:
Let V be a closed set in (Y, σ). Since f is g#s-continuous map. Therefore f⁻¹(V) is a g#s-closed in (X, τ). But every g#s-closed set is b*gα-closed set. Hence f⁻¹(V) is b*gα-closed set in (X, τ). Hence f is b*gα-continuous. Therefore every g#s-continuous map is b*gα-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.9
Let X = {a, b, c} = Y with τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {c}}, g#s C(X, τ) = {X, φ, {a}, {b, c}} and b*gα C(X, τ) = {X, φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}. Let f: (X, τ) → (Y, σ) be defined as, f(a) = a, f(b) = b and f(c) = c. Then f is b*gα-continuous but not g#s-continuous. Since f⁻¹{a, b} = {a, b} is not g#s-closed in (X, τ).

Theorem: 5.10
Every gψ-continuous map is b*gα-continuous map.

Proof:
Let V be a closed set in (Y, σ). Since f is gψ-continuous map. Therefore f⁻¹(V) is a gψ-closed in (X, τ). But every gψ-closed set is b*gα-closed set. Hence f⁻¹(V) is b*gα-closed set in (X, τ). Hence f is b*gα-continuous. Therefore every gψ-continuous map is b*gα-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.11
Let X = {a, b, c} = Y with τ = {X, φ, {a}}, σ = {Y, φ, {b}}, gψ C(X, τ) = {X, φ, {b}, {c}, {b, c}} and b*gα C(X, τ) = {X, φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}. Let f: (X, τ) → (Y, σ) be defined as, f(a) = c, f(b) = b and f(c) = a. Then f is b*gα-continuous but not gψ-continuous. Since f⁻¹{a, c} = {a, c} is not gψ-closed in (X, τ).

Theorem: 5.12
Every g-continuous map is b*gα-continuous map.

Proof:
Let V be a closed set in (Y, σ). Since f is g-continuous map. Therefore f⁻¹(V) is a g-closed in (X, τ). But every g-closed set is b*gα-closed set. Hence f⁻¹(V) is b*gα-closed set in (X, τ). Hence f is b*gα-continuous. Therefore every g-continuous map is b*gα-continuous map.

The converse of the above theorem need not be true by the following example.

Example: 5.13
Let X = {a, b, c} = Y with τ = {X, φ, {a}}, σ = {Y, φ, {b}}, g C(X, τ) = {X, φ, {b, c}} and b*gα C(X, τ) = {X, φ, {a}, {b, c}, {a, b}, {b, c}, {a, c}}. Let f: (X, τ) → (Y, σ) be defined as, f(a) = a, f(b) = b and f(c) = c. Then f is b*gα-continuous but not g-continuous. Since f⁻¹{a, c} = {a, c} is not g-closed in (X, τ).

Theorem: 5.14
i) Every αgα-continuous map is b*gα-continuous map.
ii) Every s'gα-continuous map is b*gα-continuous map.

Proof:
i) Let V be a closed set in (Y, σ). Since f is αgα-continuous map. Therefore f⁻¹(V) is a αgα-closed in (X, τ). But every αgα-closed set is b*gα-closed set. Hence f⁻¹(V) is b*gα-closed set in (X, τ). Hence f is b*gα-continuous. Therefore every αgα-continuous map is b*gα-continuous map.

ii) Let V be a closed set in (Y, σ). Since f is s'gα-continuous map. Therefore f⁻¹(V) is a s'gα-closed in (X, τ). But every s'gα-closed set is b*gα-closed set. Hence f⁻¹(V) is b*gα-closed set in (X, τ). Hence f is b*gα-continuous. Therefore every s'gα-continuous map is b*gα-continuous map.

The converse of the above theorems need not be true by the following example.
Example: 5.15

Let \( X = \{a, b, c\} = Y \) with \( \tau = \{X, \phi, \{a, b\}\} \), \( \sigma = \{Y, \phi, \{a, c\}\} \). 
\[ \forall \alpha \in \mathcal{C}(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\} \]
\[ s'_{\alpha} \mathcal{C}(X, \tau) \text{ and } b'_{\alpha} \mathcal{C}(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b, c\}\}. \]
\[ \text{Let } f: (X, \tau) \to (Y, \sigma) \text{ be defined as, } f(a) = a, f(b) = b \text{ and } f(c) = c. \]
Then \( f \) is \( b'_{\alpha} \)-continuous but not \( \alpha_{\phi} \)-continuous and \( s'_{\alpha} \)-continuous. Since \( f^{-1}\{b\} = \{b\} \) is not \( \alpha_{\phi} \)-closed and \( s'_{\alpha} \)-closed in \( (X, \tau) \).

We introduce the following definition.

Definition: 5.16

A function \( f: (X, \tau) \to (Y, \sigma) \) is called \( b^*_{\alpha} \)-irresolute if \( f^{-1}(V) \) is a \( b^*_{\alpha} \)-closed set of \( (X, \tau) \) for every \( b^*_{\alpha} \)-closed set of \( (Y, \sigma) \).

Theorem: 5.17

Every \( b^*_{\alpha} \)-irresolute function is \( b^*_{\alpha} \)-continuous.

Proof:

Let \( V \) be a closed set in \((Y, \sigma)\). Since every closed set is \( b^*_{\alpha} \)-closed set. Therefore \( V \) is \( b^*_{\alpha} \)-closed set of \((Y, \sigma)\).
Since \( f \) is \( b^*_{\alpha} \)-irresolute, \( f^{-1}(V) \) is \( b^*_{\alpha} \)-closed set in \((X, \tau)\). Therefore \( f \) is \( b^*_{\alpha} \)-continuous.

Example: 5.18

Let \( X = \{a, b, c\} = Y \) with \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}, \sigma = \{Y, \phi, \{a, c\}, \{a, b, c\}\} \). \( b^*_{\alpha} \mathcal{C}(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{a, b, c\}\} \).
\[ \text{Let } f: (X, \tau) \to (Y, \sigma) \text{ be defined as, } f(a) = b, f(b) = a \text{ and } f(c) = c. \]
Here \( f \) is \( b^*_{\alpha} \)-continuous but \( f \) is not \( b^*_{\alpha} \)-irresolute. Since \( \{a, b\} \) is \( b^*_{\alpha} \)-closed set in \((Y, \sigma)\) but \( f^{-1}\{a, b\} = \{a, b\} \) is not \( b^*_{\alpha} \)-closed set in \((X, \tau)\).

Theorem: 5.19

Let \( (X, \tau), (Y, \sigma) \) and \((Z, \eta)\) be any three topological spaces. Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \eta) \) be any two functions. Then,

i) \( g \circ f: (X, \tau) \to (Z, \eta) \) is \( b^*_{\alpha} \)-continuous if \( g \) is continuous and \( f \) is \( b^*_{\alpha} \)-continuous.

ii) \( g \circ f: (X, \tau) \to (Z, \eta) \) is \( b^*_{\alpha} \)-continuous if \( g \) is \( b^*_{\alpha} \)-continuous and \( f \) is \( b^*_{\alpha} \)-continuous.

iii) \( g \circ f: (X, \tau) \to (Z, \eta) \) is \( b^*_{\alpha} \)-irresolute if both \( g \) and \( f \) are \( b^*_{\alpha} \)-irresolute.

Proof:

i) Let \( V \) be a closed set in \((Z, \eta)\). Since \( g \) is continuous, \( g^{-1}(V) \) is closed in \((Y, \sigma)\). Since \( f \) is \( b^*_{\alpha} \)-continuous, \( f^{-1}(g^{-1}(V)) = g \circ f^{-1}(V) \) is \( b^*_{\alpha} \)-closed in \((X, \tau)\). Therefore \( g \circ f \) is \( b^*_{\alpha} \)-continuous.

ii) Let \( V \) be a closed set in \((Z, \eta)\). Since \( g \) is \( b^*_{\alpha} \)-continuous, \( g^{-1}(V) \) is \( b^*_{\alpha} \)-closed in \((Y, \sigma)\). Since \( f \) is \( b^*_{\alpha} \)-irresolute, \( f^{-1}(g^{-1}(V)) = g \circ f^{-1}(V) \) is \( b^*_{\alpha} \)-closed in \((X, \tau)\). Therefore \( g \circ f \) is \( b^*_{\alpha} \)-continuous.

iii) Let \( V \) be a \( b^*_{\alpha} \)-closed set in \((Z, \eta)\). Since \( g \) and \( f \) are \( b^*_{\alpha} \)-irresolute, \( f^{-1}(g^{-1}(V)) \) is \( b^*_{\alpha} \)-closed in \((X, \tau)\). Therefore \( g \circ f \) is \( b^*_{\alpha} \)-continuous.

VI. \( b^*_{\alpha} \)-OPEN MAPS and \( b^*_{\alpha} \)-HOMEOMORPHISMS

Definition: 6.1

A map \( f: (X, \tau) \to (Y, \sigma) \) is called a \( b^*_{\alpha} \)-open map if \( f(U) \) is \( b^*_{\alpha} \)-open in \((Y, \sigma)\) for every open set \( U \) of \((X, \tau)\).

Definition: 6.2

A map \( f: (X, \tau) \to (Y, \sigma) \) is called a \( b^*_{\alpha} \)-closed map if \( f(U) \) is \( b^*_{\alpha} \)-closed in \((Y, \sigma)\) for every closed set \( U \) of \((X, \tau)\).

Definition: 6.3

A map \( f: (X, \tau) \to (Y, \sigma) \) is called a \( b^*_{\alpha} \)-homeomorphism if \( f \) is \( b^*_{\alpha} \)-continuous and \( b^*_{\alpha} \)-open.
**Theorem 6.4**

Every open map is a $b^*g_x$-open map.

**Proof:**

Let $f: (X, \tau) \to (Y, \sigma)$ be an open map. Let $U$ be an open set in $(X, \tau)$. Since $f$ is an open map, therefore $f(U)$ is open set in $(Y, \sigma)$. Since every open set is a $b^*g_x$-open set in $(Y, \sigma)$. Then $f(U)$ is $b^*g_x$-open in $(Y, \sigma)$. Hence $f$ is a $b^*g_x$-open map.

**Theorem 6.5**

Every $g_x$-open map is a $b^*g_x$-open map.

**Proof:**

Let $f: (X, \tau) \to (Y, \sigma)$ be an $g_x$-open map. Let $U$ be an open set in $(X, \tau)$. Since $f$ is an $g_x$-open map, therefore $f(U)$ is $g_x$-open set in $(Y, \sigma)$. Since every $g_x$-open set is a $b^*g_x$-open set in $(Y, \sigma)$. Then $f(U)$ is $b^*g_x$-open in $(Y, \sigma)$. Hence $f$ is a $b^*g_x$-open map.

**Remark 6.6**

- Every $g^2x$-open map is a $b^*g_x$-open map.
- Every $g^s$-open map is a $b^*g_x$-open map.
- Every $g^\Psi$-open map is a $b^*g_x$-open map.
- Every $g^\alpha$-open map is a $b^*g_x$-open map.
- Every $s^\alpha$-open map is a $b^*g_x$-open map.

**Theorem 6.7**

Every $b^*g_x$-open map is a gsp-open map.

**Proof:**

Let $f: (X, \tau) \to (Y, \sigma)$ be an $b^*g_x$-open map. Let $U$ be an open set in $(X, \tau)$. Since $f$ is an $b^*g_x$-open map, therefore $f(U)$ is $b^*g_x$-open set in $(Y, \sigma)$. Since every $b^*g_x$-open set is a gsp-open set in $(Y, \sigma)$. Then $f(U)$ is gsp-open in $(Y, \sigma)$. Hence $f$ is a gsp-open map.

**Theorem 6.8**

Every homeomorphism is a $b^*g_x$-homeomorphism.

**Proof:**

Let $f$ be a homeomorphism from a topological space $(X, \tau)$ into a topological space $(Y, \sigma)$. Since every continuous map is $b^*g_x$-continuous and every open map is $b^*g_x$-open map, we conclude that $f$ is a $b^*g_x$-homeomorphism.

**Theorem 6.9**

Every $g_x$-homeomorphism is a $b^*g_x$-homeomorphism.

**Proof:**

Let $f$ be a $g_x$-homeomorphism from a topological space $(X, \tau)$ into a topological space $(Y, \sigma)$. Since every $g_x$-continuous map is $b^*g_x$-continuous and every $g_x$-open map is $b^*g_x$-open map, we conclude that $f$ is a $b^*g_x$-homeomorphism.

**Remark 6.10**

- Every $g^2x$- homeomorphism is a $b^*g_x$- homeomorphism.
- Every $g^s$- homeomorphism is a $b^*g_x$- homeomorphism.
- Every $g^\Psi$- homeomorphism is a $b^*g_x$- homeomorphism.
- Every $g^\alpha$- homeomorphism is a $b^*g_x$- homeomorphism.
- Every $s^\alpha$- homeomorphism is a $b^*g_x$- homeomorphism.
- Every $s^\alpha$- homeomorphism is a $b^*g_x$- homeomorphism.
Theorem 6.11
Every b*gα-homeomorphism is a gsp-homeomorphism.

Proof:
Let f be a b*gα-homeomorphism from a topological space (X, τ) into a topological space (Y, σ). Since every b*gα-continuous map is gsp-continuous and every b*gα-open map is gsp-open map, we conclude that f is a gsp-homeomorphism.

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